

Overview

I highlight here three essential aspects of this book's nonstandard approach to group theory, and briefly discuss its organization.

First and foremost, images and visual examples are the heart of this book. There are more than 300 images, an average of more than one per page. The most used visualization tool is Cayley diagrams (defined in Chapter 2) because they represent group structure clearly and faithfully. But multiplication tables and objects with symmetry also appear regularly, and to a lesser extent cycle graphs, Hasse diagrams, action diagrams, homomorphism diagrams, and more. As you can tell by flipping through the pages, visualization is the name of the game.

Secondly, I focus more on finite groups than on infinite ones. This is partially because they are easier to diagram, but more so because they give a solid foundation of intuition for group theory in general. Understanding finite groups well makes the infinite a natural generalization. This approach sacrifices little, if anything, because so much remains to study in the realm of the finite. I cover the most common infinite groups, and each chapter's exercises includes some on infinite groups.

Lastly, this book approaches groups from the opposite direction of many traditional textbooks. The usual paradigm defines a group as a set with a binary operation, and later proves Cayley's Theorem, that every group is a collection of permutations (or you could say that every group acts on some set, most notably itself). The traditional definition does not appear in this book until Chapter 4; I define groups in Chapter 1 as collections of actions, and later prove that they can also be thought of as sets with binary operations. This nonstandard paradigm facilitates my introduction in Chapter 2 of Cayley diagrams, which depict groups as collections of actions.

The book's structure is linear, to be read in order; later sections usually depend on earlier ones. There are two exceptions. Chapter 5, which gives broad exposure to finite groups, is helpful but not strictly necessary for what follows. You could skip most of it (except the definition of abelian in Section 5.2) and turn back to it later as needed. The other exception is that Chapter 10 depends only slightly on Chapter 9; Cauchy's Theorem from Section 9.2 is used in Section 10.7, and the remainder of Chapter 9 may be useful in a few of the exercises in Chapter 10.

Chapter 10, on Galois theory, aims to show the power of group theory and some of its historical roots. It includes an introduction to fields, but several theorems are stated without proof. It gives enough understanding to see how group theory ties in, and points the reader elsewhere for more details on fields. The beautiful and historic result of the unsolvability of the quintic is the focus and endpoint of that final chapter.